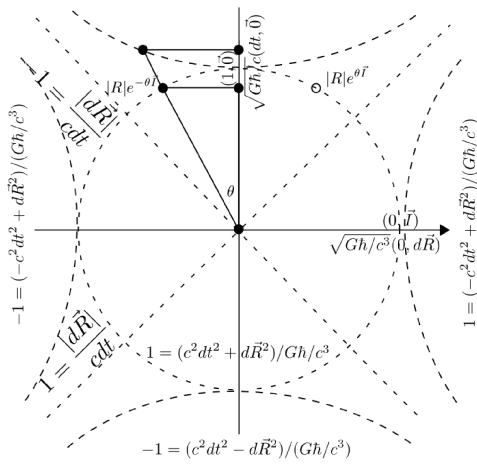


Spacetime Trig Functions Graph

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This one-page figure shows the relationships between sin/cos/tan and their hyperbolic counterparts for events in spacetime. The standard trig functions are the three pairwise ratios: sine -> far/hypotenuse, cosine -> near/hypotenuse, tangent -> far/near. What is not often known is the hyperbolics are even simpler, being only about one side of a trinance made to the hyperbola: sine -> far, cosine -> near, tangent -> hypotenuse. Any point in spacetime can be plotted using Euclidean, polar (scaled signs and cosines) or hyperbolic coordinates (scaled hyperbolic sines and cosines).

Spacetime Trig Functions



Distance & Velocities

$$\nabla \quad d = |dR| \quad \nabla \quad d = \sinh(\theta)$$

$$s = \frac{1}{c} \frac{dR}{dt} = \beta \quad \nabla \quad = \gamma\beta = \frac{\beta}{\sqrt{1-\beta^2}}$$

$$\nabla \quad d = c|dt| \quad \nabla \quad d = \cosh(\theta)$$

$$s = \frac{1}{c} \frac{dR}{dt} = \beta \quad \nabla \quad = \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\nabla \quad d = \sqrt{c^2 dt^2 + dR^2} \quad \nabla \quad d = \sqrt{\cosh(2\theta)}$$

$$s = \frac{1}{c} \frac{dR}{dt} = \beta \quad \nabla \quad = \frac{\sqrt{1+\beta^2}}{\sqrt{1-\beta^2}}$$

Ratios

$$\nabla \quad \nabla \quad \sin(\theta) = \frac{dR}{\sqrt{c^2 dt^2 + dR^2}}$$

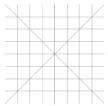
$$= \frac{1}{\sqrt{\beta^2 + 1}}$$

$$\nabla \quad \nabla \quad \cos(\theta) = \frac{c dt}{\sqrt{c^2 dt^2 + dR^2}}$$

$$= \frac{1}{\sqrt{1 + \beta^2}}$$

$$\nabla \quad \nabla \quad \tan(\theta) = \frac{1}{c} \frac{dR}{dt} = \beta$$

A unit circle, parabola, & lightcone



Euclidean



Polar Coordinates

Hyperbolic

$$\text{Sine} \quad (e^{(dt, \vec{0})/\sqrt{Gh/c}} - e^{(-dt, \vec{0})/\sqrt{Gh/c}})/2$$

$$\text{Cosine} \quad (e^{(dt, \vec{0})/\sqrt{Gh/c}} + e^{(-dt, \vec{0})/\sqrt{Gh/c}})/2$$

Trigonometric

$$(e^{(0, \vec{R})/\sqrt{Gh/c^3}} - e^{(0, \vec{R})^*/\sqrt{Gh/c^3}})/2$$

$$(e^{(0, \vec{R})/\sqrt{Gh/c^3}} + e^{(0, \vec{R})^*/\sqrt{Gh/c^3}})/2$$