Spacetime Trig Functions Graph

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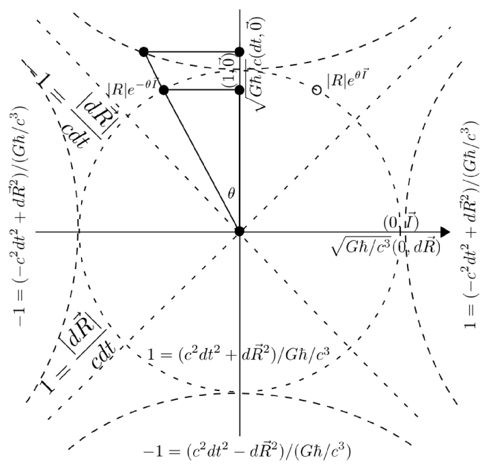
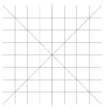
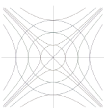
This one-page figure shows the relationships between sin/cos/tan and their hyperbolic counterparts for events in spacetime. The standard trig functions are the three pairwise ratios: sine -> far/hypothenuse, cosine -> near/hypothenuse, tangent -> far/near. What is not often known is the hyperbolics are even simpler, being only about one side of a trinage made to the hyperbola: sine -> far, cosine -> near, tanget -> hypothenuse. Any point in spacetime can be plotted using Euclidean, polar (scaled signs and cosines) or hyberbolic coordinates (scaled hyperbolic sines and cosines.

Spacetime Trig Functions



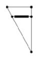
A unit circle, parabola, & lightcone


Euclidean

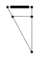

Polar Coordinates

	Hyperbolic	Trigonometric
Sine	$(e^{(dt,\vec{0})/\sqrt{G\hbar/c}}-e^{(-dt,\vec{0})/\sqrt{G\hbar/c}})/2$	$(e^{(0,\vec{R})/\sqrt{G\hbar/c^3}}-e^{(0,\vec{R})^*/\sqrt{G\hbar/c^3}})/2$
Cosine	$(e^{(dt,\vec{0})/\sqrt{G\hbar/c}}+e^{(-dt,\vec{0})/\sqrt{G\hbar/c}})/2$	$(e^{(0,\vec{R})/\sqrt{G\hbar/c^3}}+e^{(0,\vec{R})^*/\sqrt{G\hbar/c^3}})/2$

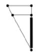
Distance & Velocities


$d=|dR|$

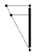
$s=\frac{1}{c}\frac{d\vec{R}}{dt}=\infty$


$d=sinh(\theta)$

$=\gamma\beta=\frac{\beta}{\sqrt{1-\beta^2}}$


$d=c|dt|$

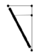
$s=\frac{1}{c}\frac{d\vec{R}}{dt}=\vec{0}$


$d=cosh(\theta)$

$=\gamma=\frac{1}{\sqrt{1-\beta^2}}$

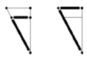

$d=\sqrt{c^2dt^2+dR^2}$

$s=\frac{1}{c}\frac{d\vec{R}}{dt}=\vec{\beta}$

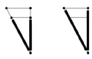

$d=\sqrt{cosh(2\theta)}$

$=\frac{\sqrt{1+\beta^2}}{\sqrt{1-\beta^2}}$

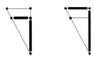
Ratios


$sin(\theta)=\frac{dR}{\sqrt{c^2dt^2+dR^2}}$

$=\frac{1}{\sqrt{\beta^{-2}+1}}$


$cos(\theta)=\frac{cdt}{\sqrt{c^2dt^2+dR^2}}$

$=\frac{1}{\sqrt{1+\beta^2}}$


$tan(\theta)=\frac{1}{c}\frac{d\vec{R}}{dt}=\vec{\beta}$