DRAFT: Specific speculations on the Strong Extremes of Gravity

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Strong field gravity must involve a metric that has Planck's constant. The GEM field equations have G, $\hbar$, and c, so could do the job based on the units. Potential solutions are found of the Maxwell equations in the Lorenz gauge, the hypercomplex gravity equations in the Lorenz gauge, and the gauge-invariant GEM field equations. Metrics are proposed to also solve the field equations. The Christoffel symbols are calculated for each of these metrics. The metrics do not solve the field equations unless an *ad hoc* but precise rule is applied. I suspect I may need to abandon the accounting system used by Riemann so long ago, a scary idea.

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Strong Gravity

The singular solutions for strong field gravity were initially known as frozen stars. Einstein considered them a flaw in general relativity that needed to be solved. When re-branded as black holes by Wheeler, the flaw has been embraced by most professionals. One could devote a career to the subject.

Astronomers have shown very small areas of the sky have very large masses. That observation will remain no matter what mathematical physics is used to describe such a location. It is a commonly held view that when one gets closer to the singularity, effects of quantum gravity will come into play. At this time, we do not have a quantum gravity theory, so people argue this makes the work interesting.

It is simple enough to spot a quantum gravity theory: it has Newton's gravitational constant G and Planck's constant $\hbar$. Until one has those two together in a way that is consistent with current observations of gravity, all work with areas of high density will be discarded by the quantum gravity theory. Like a market bubble, the value of the work done on black holes is destine to collapse and disappear into a frozen star.

The gauge-invariant GEM unified field equation looks like this:

$$
\begin{aligned}
0 &= \frac{G\hbar}{c^3}\nabla^2\phi \\
0 &= \frac{G\hbar}{c^3}\left(\frac{\partial A_i}{c\,\partial t} - \frac{\partial A_i}{\partial x_j} - \frac{\partial A_i}{\partial x_k}\right)
\end{aligned}
\tag{1}
$$

The units of the potential must be dimensionless to represent the group *U(1)*, a normalized complex number. Normalization will wipe out the units. The group *U(1)* normalization requirement helps explain why the potential cannot be directly measured: no matter how big or small the potential is, Nature normalizes it to unity in her accounting system. Calculate the size of this area:

calc

$$\frac{G\hbar}{c^3} \text{ /. Units}$$

calc

$$L^2$$

calc

```
<< PhysicalConstants`
```

calc

```
Simplify[GravitationalConstant PlanckConstantReduced / SpeedOfLight^3]
```

calc

$$\frac{2.61227\times10^{-70}\ \text{Joule Newton Second}^4}{\text{Kilogram}^2\ \text{Meter}}$$

$$G \hbar c^{-3} \left(\frac{L^3}{M T^2}\right) \left(\frac{M L^2}{T}\right) \left(\frac{T^3}{L^3}\right) = 2.6 \text{ x } 10^{-70} \text{ m}^2 \qquad (2)$$

These equations use the Planck length squared to make the field equation squared, a factor of $\sim 2.6 \text{ x} 10^{-70} m^2$. I have no idea why this area happens to be so small.

The practical approach to gravity is Newton's law, which will only have the constant G. A more refined version is a relativistic approach and also has the speed of light c. So what is involved in the transition from the GEM field equations to these more practical forms? Use the practical approach of units. Consider Newton's force law:

calc

$$\mathbf{G\, m \nabla \phi \;/.\; Units \;/.\; \phi \to \frac{M}{L}}$$

calc

$$\frac{L^2 M \nabla}{T^2}$$

$$-G\, m \nabla \phi \left(\frac{L^3}{M T^2}\right) (M) \left(\frac{1}{L}\right) \left(\frac{M}{L}\right) = F \left(\frac{M L}{T^2}\right) \qquad (3)$$

There are choices in units where mass can be measured as a length. With such a choice, the potential would remain dimensionless, a check of consistency. This equation is classical gravity because it only has the constant G. Let's make this expression relativistic by substituting energy per unit length in for an invariant mass:

calc

$$\mathbf{G\, m \nabla \phi / C^2 \;/.\; Units \;/.\; \phi \to \frac{M L}{T^2}}$$

calc

$$\frac{L^2 M \nabla}{T^2}$$

$$-G\, m\, \nabla \phi / c^2 \left(\frac{L^3}{M T^2}\right) (M) \left(\frac{1}{L}\right) \left(\frac{M L}{T^2}\right) \left(\frac{T^2}{L^2}\right) = F \left(\frac{M L}{T^2}\right) \qquad (4)$$

This expression is superficially relativistic gravity based on the units. A more complete analysis has a Lorentz-like force with a symmetric curl and a non-zero scalar term.

The following equations have full exact solutions:

1 - The Maxwell equations in the Lorenz gauge

2 - The hypercomplex G field equations in the Lorenz gauge

3 - The unified GEM field equations which are independent of gauge choice.

Do these one at a time. Assume a completely flat, Minkowski background, and find the potential that solves the field equations.

Sidebar: For those well-trained in the art of general relativity, they may stop upon reading of such a condition on the metric. General relativity is all about determining what the dynamic metric *is*, there being no potential. It brings to mind a serious critique of work on strings, that it too assumes a flat, Minkowski background metric, and therefore is in conflict with much experimental data. What the GEM proposal claims is that all potential solutions for this work can be written in a metric form. Here is the heart of the trick. Say we find a potential is a solution to a fix background. Presume a constant potential so a solution comes exclusively from the metric. Form a metric from the potential solution like so:

$$m = \text{diag}\left(\exp(2\phi), -\exp(2 A_x), -\exp\left(2 A_y\right), -\exp(2 A_z)\right) \qquad (5)$$

Calculate the Christoffel for this metric. That involves taking 3 derivatives of this metric. The derivative of an exponential is the exponential times the derivative of the exponent. The exponent is already known to solve the field equations. In the Christoffel symbol, the inverse of the metric will cancel with the metric, so all that remains is the derivative of the potential which is a solution. I have done this for a static metric, and at the end of this notebook, we will see if it works for all the cases discussed below. Rest assured, all potential solutions will be written as metrics and tested to see if they solve the same field equations. [end of sidebar]

Start with the vacuum Maxwell equations in the Lorenz gauge:

calc

```
emlorentguagefieldeq[A_] := D[A, {t, 2}] - D[A, {x, 2}] - D[A, {y, 2}] - D[A, {z, 2}]
```

$$\frac{\partial^2 A_i}{\partial t^2} - \nabla^2 A_i = 0 \tag{6}$$

Find a potential that solves one, and you have the others:

calc

```
emlorentguagefieldeq[1/(t^2 - x^2 - y^2 - z^2)]
emlorentguagefieldeq[1/(t^2 - x^2 - y^2 - z^2)] // Simplify
```

calc

$$\frac{8 t^2}{(t^2 - x^2 - y^2 - z^2)^3} - \frac{8 x^2}{(t^2 - x^2 - y^2 - z^2)^3} - \frac{8 y^2}{(t^2 - x^2 - y^2 - z^2)^3} - \frac{8 z^2}{(t^2 - x^2 - y^2 - z^2)^3} - \frac{8}{(t^2 - x^2 - y^2 - z^2)^2}$$

calc

0

$$\begin{aligned} A_i &= \frac{1}{t^2 - x^2 - y^2 - z^2} = \frac{1}{\tau^2} \\ \frac{\partial^2 A_i}{\partial t^2} - \nabla^2 A_i &= \frac{8 t^2}{\tau^6} - \frac{8 x^2}{\tau^6} - \frac{8 y^2}{\tau^6} - \frac{8 z^2}{\tau^6} - \frac{8}{\tau^4} = 0 \end{aligned} \tag{7}$$

If one takes the derivative of the potential and drops that into a force equation, it results in an inverse cube law, the stuff of dipoles.

All that changes for the hypercomplex gravity equations is a few signs.

calc

```
glorentguagefieldeq0[A_] := D[A, {t, 2}] + D[A, {x, 2}] + D[A, {y, 2}] + D[A, {z, 2}]
glorentguagefieldeq1[A_] := -D[A, {t, 2}] - D[A, {x, 2}] + D[A, {y, 2}] + D[A, {z, 2}]
glorentguagefieldeq2[A_] := -D[A, {t, 2}] + D[A, {x, 2}] - D[A, {y, 2}] + D[A, {z, 2}]
glorentguagefieldeq3[A_] := -D[A, {t, 2}] + D[A, {x, 2}] + D[A, {y, 2}] - D[A, {z, 2}]
```

$$\begin{aligned} &\frac{\partial^2 \phi}{\partial t^2} + \nabla^2 \phi = 0 \\ &-\frac{\partial^2 A_i}{\partial t^2} - \frac{\partial^2 A_i}{\partial {x_i}^2} + \frac{\partial^2 A_i}{\partial {x_j}^2} + \frac{\partial^2 A_i}{\partial {x_k}^2} = 0 \end{aligned} \tag{8}$$

Because the field equations for the Maxwell equations in the Lorentz gauge were in a sense degenerate - all the same - one solution fit all. Here the equations are all variations on those. Put in the minus sign at the right location, and solutions are found.

calc

```
glorentguagefieldeq0[1/(t^2 + x^2 + y^2 + z^2)]
glorentguagefieldeq0[1/(t^2 + x^2 + y^2 + z^2)] // Simplify
```

calc

$$\frac{8 t^2}{(t^2 + x^2 + y^2 + z^2)^3} + \frac{8 x^2}{(t^2 + x^2 + y^2 + z^2)^3} + \frac{8 y^2}{(t^2 + x^2 + y^2 + z^2)^3} + \frac{8 z^2}{(t^2 + x^2 + y^2 + z^2)^3} - \frac{8}{(t^2 + x^2 + y^2 + z^2)^2}$$

calc

0

calc

```
glorentguagefieldeq1[1/(t^2 + x^2 - y^2 - z^2)]
glorentguagefieldeq1[1 / (t^2 + x^2 - y^2 - z^2)] // Simplify
```

calc

$$-\frac{8\,t^2}{\left(t^2+x^2-y^2-z^2\right)^3}-\frac{8\,x^2}{\left(t^2+x^2-y^2-z^2\right)^3}+\frac{8\,y^2}{\left(t^2+x^2-y^2-z^2\right)^3}+\frac{8\,z^2}{\left(t^2+x^2-y^2-z^2\right)^3}+\frac{8}{\left(t^2+x^2-y^2-z^2\right)^2}$$

calc

0

$$\phi=\frac{1}{t^2+x^2+y^2+z^2}=\frac{1}{\|\,t,\,R\,\|}$$
$$\frac{\partial^2\phi}{\partial t^2}+\nabla^2\phi=\frac{8\,t^2}{[\![t,\,R]\!]^3}+\frac{8\,x^2}{[\![t,\,R]\!]^3}+\frac{8\,y^2}{[\![t,\,R]\!]^3}+\frac{8\,z^2}{[\![t,\,R]\!]^3}-\frac{8}{[\![t,\,R]\!]^2}=0$$
$$A_i=\frac{1}{t^2+x_i{}^2-x_j{}^2-x_k{}^2}\equiv\frac{1}{\kappa^2} \tag{9}$$
$$-\frac{\partial^2 A_i}{\partial t^2}-\frac{\partial^2 A_i}{\partial t^2}+\frac{\partial^2 A_i}{\partial x_j{}^2}+\frac{\partial^2 A_i}{\partial x_k{}^2}=-\frac{8\,t^2}{\kappa^6}-\frac{8\,x^2}{\kappa^6}+\frac{8\,y^2}{\kappa^6}+\frac{8\,z^2}{\kappa^6}+\frac{8}{\kappa^4}=0$$

If one forms a force equation by taking a derivative of any of these potentials, then one gets a distance-like cubed term.

calc

```
GEMfieldeq0[A_] := D[A, {x, 2}] + D[A, {y, 2}] + D[A, {z, 2}]
GEMfieldeq1[A_] := -D[A, {t, 2}] + D[A, {y, 2}] + D[A, {z, 2}]
GEMfieldeq2[A_] := -D[A, {t, 2}] + D[A, {x, 2}] + D[A, {z, 2}]
GEMfieldeq3[A_] := -D[A, {t, 2}] + D[A, {x, 2}] + D[A, {y, 2}]
```

$$\nabla^2\phi=0$$
$$-\frac{\partial^2 A_i}{\partial t^2}+\frac{\partial^2 A_i}{\partial x_j{}^2}+\frac{\partial^2 A_i}{\partial x_k{}^2}=0 \tag{10}$$

Solutions to the gauge-invariant unified GEM field are similar, but have 1/distance potentials instead of 1/distance squared. Since the Poisson 1/R solution is the best known in physics, look at one of the 3-vector potentials:

calc

```
Ax = 1/Sqrt[t^2 - y^2 - z^2];
GEMfieldeq1[Ax]
GEMfieldeq1[Ax] // Simplify
```

calc

$$-\frac{3\,t^2}{\left(t^2-y^2-z^2\right)^{5/2}}+\frac{3\,y^2}{\left(t^2-y^2-z^2\right)^{5/2}}+\frac{3\,z^2}{\left(t^2-y^2-z^2\right)^{5/2}}+\frac{3}{\left(t^2-y^2-z^2\right)^{3/2}}$$

calc

0

$$A_x=\frac{1}{\sqrt{t^2-y^2-z^2}}\equiv\frac{1}{\tau_{-x}}$$
$$-\frac{\partial^2 A_i}{\partial t^2}+\frac{\partial^2 A_i}{\partial x_j{}^2}+\frac{\partial^2 A_i}{\partial x_k{}^2}=-\frac{3\,t^2}{\tau_{-x}{}^5}+\frac{3\,y^2}{\tau_{-x}{}^5}+\frac{3\,z^2}{\tau_{-x}{}^5}+\frac{3}{\tau_{-x}{}^3}=0 \tag{11}$$

A similar solution can be found for the *y* and *z* cases. After some consideration, the symbol chosen was tau without x, τ_{-x}. This is not as novel as it may first appear, since one can view the radius as minus tau without t. Each of the four solutions to the unified GEM field equations uses its own tau without the dimension that would be in a gauge term.

Now see if these potential solutions can all be viewed as metric solutions. Start with Maxwell. There have been grand efforts to "geometerize" electromagnetism. That is not the goal, which is far smaller. The question is if this exponential trick works for the Maxwell equations written in the Lorenz gauge.

calc

```
emlorenzgaugemetric =
 {{-Exp[-2/(t^2 - x^2 - y^2 - z^2)], 0, 0, 0}, {0, -Exp[-2/(t^2 - x^2 - y^2 - z^2)], 0, 0},
  {0, 0, -Exp[-2/(t^2 - x^2 - y^2 - z^2)], 0}, {0, 0, 0, Exp[2/(t^2 - x^2 - y^2 - z^2)]}}

glorenzgaugemetric = {{-Exp[2/(t^2 + x^2 - y^2 - z^2)], 0, 0, 0}, {0, -Exp[2/(t^2 - x^2 + y^2 - z^2)], 0, 0},
  {0, 0, -Exp[2/(t^2 - x^2 - y^2 + z^2)], 0}, {0, 0, 0, Exp[-2/(t^2 + x^2 + y^2 + z^2)]}};
gemmetric = {{-Exp[2/Sqrt[(t^2 - y^2 - z^2)]], 0, 0, 0}, {0, -Exp[2/Sqrt[(t^2 - x^2 - z^2)]], 0, 0},
  {0, 0, -Exp[2/Sqrt[(t^2 - x^2 - y^2)]], 0}, {0, 0, 0, Exp[-2/Sqrt[(x^2 + y^2 + z^2)]]}};
```

calc

$$\left\{\left\{-e^{-\frac{2}{t^2-x^2-y^2-z^2}}, 0, 0, 0\right\}, \left\{0, -e^{-\frac{2}{t^2-x^2-y^2-z^2}}, 0, 0\right\}, \left\{0, 0, -e^{-\frac{2}{t^2-x^2-y^2-z^2}}, 0\right\}, \left\{0, 0, 0, e^{\frac{2}{t^2-x^2-y^2-z^2}}\right\}\right\}$$

$$g_{\mathrm{EM}} = \mathrm{diag}\left(\exp\left(\frac{2G\sqrt{\frac{\hbar}{c^7}}\,q}{(t^2-x^2-y^2-z^2)}\right), -\exp\left(-\frac{2G\sqrt{\frac{\hbar}{c^7}}\,q}{c^2(t^2-x^2-y^2-z^2)}\right), -\mathrm{Exp}\left(-\frac{2G\sqrt{\frac{\hbar}{c^7}}\,q}{c^2(t^2-x^2-y^2-z^2)}\right), -\exp\left(-\frac{2G\sqrt{\frac{\hbar}{c^7}}\,q}{c^2(t^2-x^2-y^2-z^2)}\right)\right)$$

$$g_g = \mathrm{diag}\left(\exp\left(-\frac{2\sqrt{\frac{G^3\hbar}{c^7}}\,M}{(t^2+x^2+y^2+z^2)}\right), -\exp\left(\frac{2\sqrt{\frac{G^3\hbar}{c^7}}\,M}{(t^2+x^2-y^2-z^2)}\right), -\exp\left(\frac{2\sqrt{\frac{G^3\hbar}{c^7}}\,M}{(t^2-x^2+y^2-z^2)}\right), -\exp\left(\frac{2\sqrt{\frac{G^3\hbar}{c^7}}\,M}{(t^2-x^2-y^2+z^2)}\right)\right) \tag{12}$$

$$g_{\mathrm{GEM}} = \mathrm{diag}\left(\exp\left(-\frac{2(GM-\sqrt{G}\,q)}{c^2\sqrt{x^2+y^2+z^2}}\right), -\exp\left(\frac{2(GM-\sqrt{G}\,q)}{c^2\sqrt{t^2-y^2-z^2}}\right), -\exp\left(\frac{2(GM-\sqrt{G}\,q)}{c^2\sqrt{t^2-x^2-z^2}}\right), -\exp\left(\frac{2(GM-\sqrt{G}\,q)}{c^2\sqrt{t^2-x^2-y^2}}\right)\right)$$

calc

```
G Sqrt[ħ/C^7] q/L^2 /. Units
```

calc

$$\frac{\mathrm{L}\sqrt{\mathrm{L}^3\,\mathrm{M}}\sqrt{\frac{\mathrm{M\,T}^6}{\mathrm{L}^5}}}{\mathrm{M\,T}^3}$$

calc

$$\sqrt{\frac{G^3 \hbar}{C^7}}\ \frac{M}{L^2}\ \text{/. Units // FullSimplify}$$

calc

$$\frac{\sqrt{\frac{L^4}{M^2}}\ M}{L^2}$$

The constants needed to make the exponential are rather odd.

Proving the connections solve the field equations require *Mathematica* functions found in a notebook by James Hartle from his book, "Gravity: An Introduction to Einstein's General Relativity" (http://wps.aw.com/aw_hartle_gravity_1/0,6533,512496-,00.html).

calc

Set the dimensions and coordinates:

calc

```
n = 4;
coord = {x, y, z, t};
```

Calculate the inverse metric.

calc

```
inverseemlorenzgaugemetric = Inverse[emlorenzgaugemetric]
inverseglorenzgaugemetric = Inverse[glorenzgaugemetric];
inversegemmetric = Inverse[gemmetric];
```

calc

$$\left\{\left\{-e^{\frac{2}{t^2-x^2-y^2-z^2}}, 0, 0, 0\right\}, \left\{0, -e^{\frac{2}{t^2-x^2-y^2-z^2}}, 0, 0\right\}, \left\{0, 0, -e^{\frac{2}{t^2-x^2-y^2-z^2}}, 0\right\}, \left\{0, 0, 0, e^{-\frac{2}{t^2-x^2-y^2-z^2}}\right\}\right\}$$

calc

```
affineem := affineem = Table[(1 / 2) * Sum[(inverseemlorenzgaugemetric[[i, s]]) *
       (D[emlorenzgaugemetric[[s, j]], coord[[k]] ] +
         D[emlorenzgaugemetric[[s, k]], coord[[j]] ] -
         D[emlorenzgaugemetric[[j, k]], coord[[s]] ]), {s, 1, n}],
    {i, 1, n}, {j, 1, n}, {k, 1, n}]
affineg := affineg = Table[(1 / 2) * Sum[(inverseglorenzgaugemetric[[i, s]]) *
       (D[glorenzgaugemetric[[s, j]], coord[[k]] ] +
         D[glorenzgaugemetric[[s, k]], coord[[j]] ] -
         D[glorenzgaugemetric[[j, k]], coord[[s]] ]), {s, 1, n}],
    {i, 1, n}, {j, 1, n}, {k, 1, n}]
affinegem := affinegem = Table[(1 / 2) * Sum[(inversegemmetric[[i, s]]) *
       (D[gemmetric[[s, j]], coord[[k]] ] +
         D[gemmetric[[s, k]], coord[[j]] ] - D[gemmetric[[j, k]], coord[[s]] ]),
      {s, 1, n}],
    {i, 1, n}, {j, 1, n}, {k, 1, n}]
```

calc

```
listaffineem := Table[If[UnsameQ[affineem[[i, j, k]], 0],
    {ToString[Γ[i, j, k]], affineem[[i, j, k]]}] , {i, 1, n}, {j, 1, n}, {k, 1, j}]
listaffineg := Table[If[UnsameQ[affineg[[i, j, k]], 0],
    {ToString[Γ[i, j, k]], affineg[[i, j, k]]}] , {i, 1, n}, {j, 1, n}, {k, 1, j}]
listaffinegem := Table[If[UnsameQ[affinegem[[i, j, k]], 0],
    {ToString[Γ[i, j, k]], affinegem[[i, j, k]]}] , {i, 1, n}, {j, 1, n}, {k, 1, j}]
```

calc

```
TableForm[Partition[DeleteCases[Flatten[listaffineem], Null], 2],
 TableSpacing -> {2, 2}]
```

calc

$\Gamma[1,\ 1,\ 1] \quad -\frac{2\,x}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[1,\ 2,\ 1] \quad -\frac{2\,y}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[1,\ 2,\ 2] \quad \frac{2\,x}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[1,\ 3,\ 1] \quad -\frac{2\,z}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[1,\ 3,\ 3] \quad \frac{2\,x}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[1,\ 4,\ 1] \quad \frac{2\,t}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[1,\ 4,\ 4] \quad \frac{2\,e^{\frac{4}{t^2-x^2-y^2-z^2}}\,x}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[2,\ 1,\ 1] \quad \frac{2\,y}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[2,\ 2,\ 1] \quad -\frac{2\,x}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[2,\ 2,\ 2] \quad -\frac{2\,y}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[2,\ 3,\ 2] \quad -\frac{2\,z}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[2,\ 3,\ 3] \quad \frac{2\,y}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[2,\ 4,\ 2] \quad \frac{2\,t}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[2,\ 4,\ 4] \quad \frac{2\,e^{\frac{4}{t^2-x^2-y^2-z^2}}\,y}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[3,\ 1,\ 1] \quad \frac{2\,z}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[3,\ 2,\ 2] \quad \frac{2\,z}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[3,\ 3,\ 1] \quad -\frac{2\,x}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[3,\ 3,\ 2] \quad -\frac{2\,y}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[3,\ 3,\ 3] \quad -\frac{2\,z}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[3,\ 4,\ 3] \quad \frac{2\,t}{\left(t^2-x^2-y^2-z^2\right)^2}$

$\Gamma[3,\ 4,\ 4] \quad \frac{2\,e^{\frac{4}{t^2-x^2-y^2-z^2}}\,z}{\left(t^2-x^2-y^2-z^2\right)^2}$

Γ[4, 1, 1]	$\frac{2\, e^{-\frac{4}{t^2-x^2-y^2-z^2}}\, t}{\left(t^2-x^2-y^2-z^2\right)^2}$
Γ[4, 2, 2]	$\frac{2\, e^{-\frac{4}{t^2-x^2-y^2-z^2}}\, t}{\left(t^2-x^2-y^2-z^2\right)^2}$
Γ[4, 3, 3]	$\frac{2\, e^{-\frac{4}{t^2-x^2-y^2-z^2}}\, t}{\left(t^2-x^2-y^2-z^2\right)^2}$
Γ[4, 4, 1]	$\frac{2\, x}{\left(t^2-x^2-y^2-z^2\right)^2}$
Γ[4, 4, 2]	$\frac{2\, y}{\left(t^2-x^2-y^2-z^2\right)^2}$
Γ[4, 4, 3]	$\frac{2\, z}{\left(t^2-x^2-y^2-z^2\right)^2}$
Γ[4, 4, 4]	$-\frac{2\, t}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[1, 1, 1]"	$-\frac{2\, x}{\left(t^2-x^2-y^2-z^2\right)^2}$
"Γ[1, 2, 1]"	$-\frac{2\, y}{\left(t^2-x^2-y^2-z^2\right)^2}$
"Γ[1, 2, 2]"	$\frac{2\, x}{\left(t^2-x^2-y^2-z^2\right)^2}$
"Γ[1, 3, 1]"	$-\frac{2\, z}{\left(t^2-x^2-y^2-z^2\right)^2}$
"Γ[1, 3, 3]"	$\frac{2\, x}{\left(t^2-x^2-y^2-z^2\right)^2}$
"Γ[1, 4, 1]"	$\frac{2\, t}{\left(t^2-x^2-y^2-z^2\right)^2}$
"Γ[1, 4, 4]"	$\frac{2\, e^{\frac{4}{t^2-x^2-y^2-z^2}}\, x}{\left(t^2-x^2-y^2-z^2\right)^2}$
"Γ[2, 1, 1]"	$\frac{2\, y}{\left(t^2-x^2-y^2-z^2\right)^2}$
"Γ[2, 2, 1]"	$-\frac{2\, x}{\left(t^2-x^2-y^2-z^2\right)^2}$
"Γ[2, 2, 2]"	$-\frac{2\, y}{\left(t^2-x^2-y^2-z^2\right)^2}$
"Γ[2, 3, 2]"	$-\frac{2\, z}{\left(t^2-x^2-y^2-z^2\right)^2}$
"Γ[2, 3, 3]"	$\frac{2\, y}{\left(t^2-x^2-y^2-z^2\right)^2}$
"Γ[2, 4, 2]"	$\frac{2\, t}{\left(t^2-x^2-y^2-z^2\right)^2}$
"Γ[2, 4, 4]"	$\frac{2\, e^{\frac{4}{t^2-x^2-y^2-z^2}}\, y}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[3, 1, 1]" $\frac{2z}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[3, 2, 2]" $\frac{2z}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[3, 3, 1]" $-\frac{2x}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[3, 3, 2]" $-\frac{2y}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[3, 3, 3]" $-\frac{2z}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[3, 4, 3]" $\frac{2t}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[3, 4, 4]" $\frac{2\,e^{\frac{4}{t^2-x^2-y^2-z^2}}\,z}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[4, 1, 1]" $\frac{2\,e^{-\frac{4}{t^2-x^2-y^2-z^2}}\,t}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[4, 2, 2]" $\frac{2\,e^{-\frac{4}{t^2-x^2-y^2-z^2}}\,t}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[4, 3, 3]" $\frac{2\,e^{-\frac{4}{t^2-x^2-y^2-z^2}}\,t}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[4, 4, 1]" $\frac{2x}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[4, 4, 2]" $\frac{2y}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[4, 4, 3]" $\frac{2z}{\left(t^2-x^2-y^2-z^2\right)^2}$

"Γ[4, 4, 4]" $-\frac{2t}{\left(t^2-x^2-y^2-z^2\right)^2}$

Here are the rules of using Christoffel symbols as I understand them (which is not too well having not taken a formal class in the subject). The normal derivative does not transform as a tensor. A Christoffel symbol does not transform as a tensor. Put the right combinations of derivatives with the appropriate Christoffel symbols, and they do transform as a tensor. If we take the contravariant derivative of a contravariant vector, and focus on the first term, the covariant derivatives would look like this:

$$\begin{aligned}
\nabla_4 \phi &= \frac{\partial \phi}{\partial t} - \Gamma[4,4,4] - \Gamma[4,1,4] - \Gamma[4,2,4] - \Gamma[4,3,4] \\
\nabla_1 \phi &= \frac{\partial \phi}{\partial x} - \Gamma[4,4,1] - \Gamma[4,1,1] - \Gamma[4,2,1] - \Gamma[4,3,1] \\
\nabla_2 \phi &= \frac{\partial \phi}{\partial y} - \Gamma[4,4,2] - \Gamma[4,1,2] - \Gamma[4,2,2] - \Gamma[4,3,2] \\
\nabla_3 \phi &= \frac{\partial \phi}{\partial z} - \Gamma[4,4,3] - \Gamma[4,1,3] - \Gamma[4,2,3] - \Gamma[4,3,3]
\end{aligned} \tag{14}$$

Six of the sixteen gammas are equal to zero. It became clear that this metric would not solve this field equation:

$$\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi = 0 \tag{15}$$

The Christoffel symbols generate too many derivatives to work. Therefore the metric does NOT solve the field equations.

What interests me are things that are highly constrained and consistent with experiment. It all my dealings with quaternions and hypercomplex numbers, the Christoffel symbols have always struck me as too complicated (me of scant training). What I will propose here is *ad hoc*, motivated solely by making a link between the metric and the field equations in the Lorenz gauge. The rule is "two indices for the potential". It picks out the first column of the proper definition from above:

$$\nabla_4 \phi \equiv \frac{\partial \phi}{\partial t} - \Gamma[4, 4, 4] = \frac{\partial \phi}{\partial t} + \frac{2\,t}{\left(t^2 - x^2 - y^2 - z^2\right)^2}$$

$$\nabla_1 \phi = \frac{\partial \phi}{\partial x} - \Gamma[4, 4, 1] = \frac{\partial \phi}{\partial x} - \frac{2\,x}{\left(t^2 - x^2 - y^2 - z^2\right)^2}$$

$$\nabla_2 \phi = \frac{\partial \phi}{\partial y} - \Gamma[4, 4, 2] = \frac{\partial \phi}{\partial y} - \frac{2\,y}{\left(t^2 - x^2 - y^2 - z^2\right)^2}$$

$$\nabla_3 \phi = \frac{\partial \phi}{\partial z} - \Gamma[4, 4, 3] = \frac{\partial \phi}{\partial z} - \frac{2\,z}{\left(t^2 - x^2 - y^2 - z^2\right)^2}$$

If and only if phi is constant in space and time, and spacetime is flat enough so the second derivative does not involve a second set of Christoffel symbols, then the metric solves the field equations:

calc

$$\left(D\left[\frac{2\,t}{\left(t^2 - x^2 - y^2 - z^2\right)^2}, t\right] - D\left[-\frac{2\,x}{\left(t^2 - x^2 - y^2 - z^2\right)^2}, x\right] - D\left[-\frac{2\,y}{\left(t^2 - x^2 - y^2 - z^2\right)^2}, y\right] - D\left[-\frac{2\,z}{\left(t^2 - x^2 - y^2 - z^2\right)^2}, z\right]\right) \text{// Simplify}$$

calc

0

$$0 = \frac{\partial \Gamma_{EM}[\phi, \phi, t]}{\partial t} - \frac{\partial \Gamma_{EM}[\phi, \phi, x]}{\partial x} - \frac{\partial \Gamma_{EM}[\phi, \phi, y]}{\partial y} - \frac{\partial \Gamma_{EM}[\phi, \phi, z]}{\partial z} \tag{17}$$

I will respect anyone who dismisses this observation as algebraic masturbation. To argue against this being wasted efforts, the following three equations are also true:

$$\begin{aligned}
0 &= \frac{\partial \Gamma_{EM}[A_x, A_x, t]}{\partial t} - \frac{\partial \Gamma_{EM}[A_x, A_x, x]}{\partial x} - \frac{\partial \Gamma_{EM}[A_x, A_x, y]}{\partial y} - \frac{\partial \Gamma_{EM}[A_x, A_x, z]}{\partial z} \\
0 &= \frac{\partial \Gamma_{EM}[A_y, A_y, t]}{\partial t} - \frac{\partial \Gamma_{EM}[A_y, A_y, x]}{\partial x} - \frac{\partial \Gamma_{EM}[A_y, A_y, y]}{\partial y} - \frac{\partial \Gamma_{EM}[A_y, A_y, z]}{\partial z} \\
0 &= \frac{\partial \Gamma_{EM}[A_z, A_z, t]}{\partial t} - \frac{\partial \Gamma_{EM}[A_z, A_z, x]}{\partial x} - \frac{\partial \Gamma_{EM}[A_z, A_z, y]}{\partial y} - \frac{\partial \Gamma_{EM}[A_z, A_z, z]}{\partial z}
\end{aligned} \tag{18}$$

Since there are now sixteen Christoffel symbols that work productively in four equations, it feels to me to be non-trivial

This exercise can be repeated for both the hypercomplex gravity field equations in the Lorenz gauge, and the GEM field equations. It should not be surprising that this works for them also, as they appear to be variations on the Maxwell field equations.

Here are the Christoffel symbols for the hypercomplex gravity field equations:

calc

```
TableForm[Partition[DeleteCases[Flatten[listaffineg], Null], 2], TableSpacing -> {2, 2}]
```

calc

Γ[1, 1, 1]	$-\frac{2\,x}{\left(t^2+x^2-y^2-z^2\right)^2}$
Γ[1, 2, 1]	$\frac{2\,y}{\left(t^2+x^2-y^2-z^2\right)^2}$
Γ[1, 2, 2]	$-\frac{2\,e^{-\frac{2}{t^2+x^2-y^2-z^2}+\frac{2}{t^2-x^2+y^2-z^2}}\,x}{\left(t^2-x^2+y^2-z^2\right)^2}$
Γ[1, 3, 1]	$\frac{2\,z}{\left(t^2+x^2-y^2-z^2\right)^2}$
Γ[1, 3, 3]	$-\frac{2\,e^{-\frac{2}{t^2+x^2-y^2-z^2}+\frac{2}{t^2-x^2-y^2+z^2}}\,x}{\left(t^2-x^2-y^2+z^2\right)^2}$
Γ[1, 4, 1]	$-\frac{2\,t}{\left(t^2+x^2-y^2-z^2\right)^2}$
Γ[1, 4, 4]	$\frac{2\,e^{-\frac{2}{t^2+x^2-y^2-z^2}-\frac{2}{t^2+x^2+y^2+z^2}}\,x}{\left(t^2+x^2+y^2+z^2\right)^2}$
Γ[2, 1, 1]	$-\frac{2\,e^{\frac{2}{t^2+x^2-y^2-z^2}-\frac{2}{t^2-x^2+y^2-z^2}}\,y}{\left(t^2+x^2-y^2-z^2\right)^2}$
Γ[2, 2, 1]	$\frac{2\,x}{\left(t^2-x^2+y^2-z^2\right)^2}$
Γ[2, 2, 2]	$-\frac{2\,y}{\left(t^2-x^2+y^2-z^2\right)^2}$
Γ[2, 3, 2]	$\frac{2\,z}{\left(t^2-x^2+y^2-z^2\right)^2}$
Γ[2, 3, 3]	$-\frac{2\,e^{-\frac{2}{t^2-x^2+y^2-z^2}+\frac{2}{t^2-x^2-y^2+z^2}}\,y}{\left(t^2-x^2-y^2+z^2\right)^2}$
Γ[2, 4, 2]	$-\frac{2\,t}{\left(t^2-x^2+y^2-z^2\right)^2}$
Γ[2, 4, 4]	$\frac{2\,e^{-\frac{2}{t^2-x^2+y^2-z^2}-\frac{2}{t^2+x^2+y^2+z^2}}\,y}{\left(t^2+x^2+y^2+z^2\right)^2}$
Γ[3, 1, 1]	$-\frac{2\,e^{\frac{2}{t^2+x^2-y^2-z^2}-\frac{2}{t^2-x^2-y^2+z^2}}\,z}{\left(t^2+x^2-y^2-z^2\right)^2}$
Γ[3, 2, 2]	$-\frac{2\,e^{\frac{2}{t^2-x^2+y^2-z^2}-\frac{2}{t^2-x^2-y^2+z^2}}\,z}{\left(t^2-x^2+y^2-z^2\right)^2}$
Γ[3, 3, 1]	$\frac{2\,x}{\left(t^2-x^2-y^2+z^2\right)^2}$
Γ[3, 3, 2]	$\frac{2\,y}{\left(t^2-x^2-y^2+z^2\right)^2}$
Γ[3, 3, 3]	$-\frac{2\,z}{\left(t^2-x^2-y^2+z^2\right)^2}$

Γ[3, 4, 3]	$-\frac{2\,t}{\left(t^2-x^2-y^2+z^2\right)^2}$
Γ[3, 4, 4]	$\frac{2\,e^{-\frac{2}{t^2-x^2-y^2+z^2}-\frac{2}{t^2+x^2+y^2+z^2}}\,z}{\left(t^2+x^2+y^2+z^2\right)^2}$
Γ[4, 1, 1]	$-\frac{2\,e^{\frac{2}{t^2+x^2-y^2-z^2}+\frac{2}{t^2+x^2+y^2+z^2}}\,t}{\left(t^2+x^2-y^2-z^2\right)^2}$
Γ[4, 2, 2]	$-\frac{2\,e^{\frac{2}{t^2-x^2+y^2-z^2}+\frac{2}{t^2+x^2+y^2+z^2}}\,t}{\left(t^2-x^2+y^2-z^2\right)^2}$
Γ[4, 3, 3]	$-\frac{2\,e^{\frac{2}{t^2-x^2-y^2+z^2}+\frac{2}{t^2+x^2+y^2+z^2}}\,t}{\left(t^2-x^2-y^2+z^2\right)^2}$
Γ[4, 4, 1]	$\frac{2\,x}{\left(t^2+x^2+y^2+z^2\right)^2}$
Γ[4, 4, 2]	$\frac{2\,y}{\left(t^2+x^2+y^2+z^2\right)^2}$
Γ[4, 4, 3]	$\frac{2\,z}{\left(t^2+x^2+y^2+z^2\right)^2}$
Γ[4, 4, 4]	$\frac{2\,t}{\left(t^2+x^2+y^2+z^2\right)^2}$
"Γ[1, 1, 1]"	$-\frac{2\,x}{\left(t^2+x^2-y^2-z^2\right)^2}$
"Γ[1, 2, 1]"	$\frac{2\,y}{\left(t^2+x^2-y^2-z^2\right)^2}$
"Γ[1, 2, 2]"	$-\frac{2\,e^{-\frac{2}{t^2+x^2-y^2-z^2}+\frac{2}{t^2-x^2+y^2-z^2}}\,x}{\left(t^2-x^2+y^2-z^2\right)^2}$
"Γ[1, 3, 1]"	$\frac{2\,z}{\left(t^2+x^2-y^2-z^2\right)^2}$
"Γ[1, 3, 3]"	$-\frac{2\,e^{-\frac{2}{t^2+x^2-y^2-z^2}+\frac{2}{t^2-x^2-y^2+z^2}}\,x}{\left(t^2-x^2-y^2+z^2\right)^2}$
"Γ[1, 4, 1]"	$-\frac{2\,t}{\left(t^2+x^2-y^2-z^2\right)^2}$
"Γ[1, 4, 4]"	$\frac{2\,e^{-\frac{2}{t^2+x^2-y^2-z^2}-\frac{2}{t^2+x^2+y^2+z^2}}\,x}{\left(t^2+x^2+y^2+z^2\right)^2}$
"Γ[2, 1, 1]"	$-\frac{2\,e^{\frac{2}{t^2+x^2-y^2-z^2}-\frac{2}{t^2-x^2+y^2-z^2}}\,y}{\left(t^2+x^2-y^2-z^2\right)^2}$
"Γ[2, 2, 1]"	$\frac{2\,x}{\left(t^2-x^2+y^2-z^2\right)^2}$
"Γ[2, 2, 2]"	$-\frac{2\,y}{\left(t^2-x^2+y^2-z^2\right)^2}$
"Γ[2, 3, 2]"	$\frac{2\,z}{\left(t^2-x^2+y^2-z^2\right)^2}$

"Γ[2, 3, 3]"	$-\frac{2\, e^{-\frac{2}{t^2-x^2+y^2-z^2}+\frac{2}{t^2-x^2-y^2+z^2}}\, y}{\left(t^2-x^2-y^2+z^2\right)^2}$
"Γ[2, 4, 2]"	$-\frac{2\, t}{\left(t^2-x^2+y^2-z^2\right)^2}$
"Γ[2, 4, 4]"	$\frac{2\, e^{-\frac{2}{t^2-x^2+y^2-z^2}-\frac{2}{t^2+x^2+y^2+z^2}}\, y}{\left(t^2+x^2+y^2+z^2\right)^2}$
"Γ[3, 1, 1]"	$-\frac{2\, e^{\frac{2}{t^2+x^2-y^2-z^2}-\frac{2}{t^2-x^2-y^2+z^2}}\, z}{\left(t^2+x^2-y^2-z^2\right)^2}$
"Γ[3, 2, 2]"	$-\frac{2\, e^{\frac{2}{t^2-x^2+y^2-z^2}-\frac{2}{t^2-x^2-y^2+z^2}}\, z}{\left(t^2-x^2+y^2-z^2\right)^2}$
"Γ[3, 3, 1]"	$\frac{2\, x}{\left(t^2-x^2-y^2+z^2\right)^2}$
"Γ[3, 3, 2]"	$\frac{2\, y}{\left(t^2-x^2-y^2+z^2\right)^2}$
"Γ[3, 3, 3]"	$-\frac{2\, z}{\left(t^2-x^2-y^2+z^2\right)^2}$
"Γ[3, 4, 3]"	$-\frac{2\, t}{\left(t^2-x^2-y^2+z^2\right)^2}$
"Γ[3, 4, 4]"	$\frac{2\, e^{-\frac{2}{t^2-x^2-y^2+z^2}-\frac{2}{t^2+x^2+y^2+z^2}}\, z}{\left(t^2+x^2+y^2+z^2\right)^2}$
"Γ[4, 1, 1]"	$-\frac{2\, e^{\frac{2}{t^2+x^2-y^2-z^2}+\frac{2}{t^2+x^2+y^2+z^2}}\, t}{\left(t^2+x^2-y^2-z^2\right)^2}$
"Γ[4, 2, 2]"	$-\frac{2\, e^{\frac{2}{t^2-x^2+y^2-z^2}+\frac{2}{t^2+x^2+y^2+z^2}}\, t}{\left(t^2-x^2+y^2-z^2\right)^2}$
"Γ[4, 3, 3]"	$-\frac{2\, e^{\frac{2}{t^2-x^2-y^2+z^2}+\frac{2}{t^2+x^2+y^2+z^2}}\, t}{\left(t^2-x^2-y^2+z^2\right)^2}$
"Γ[4, 4, 1]"	$\frac{2\, x}{\left(t^2+x^2+y^2+z^2\right)^2}$
"Γ[4, 4, 2]"	$\frac{2\, y}{\left(t^2+x^2+y^2+z^2\right)^2}$
"Γ[4, 4, 3]"	$\frac{2\, z}{\left(t^2+x^2+y^2+z^2\right)^2}$
"Γ[4, 4, 4]"	$\frac{2\, t}{\left(t^2+x^2+y^2+z^2\right)^2}$

There are more Christoffel symbols that have exponentials for the hypercomplex gravity metric than for the EM metric because of the different potentials that are in the gravity metric.

calc

$$\left(D\left[\frac{2\, t}{\left(t^2+x^2+y^2+z^2\right)^2},\, t\right]+D\left[\frac{2\, x}{\left(t^2+x^2+y^2+z^2\right)^2},\, x\right]+D\left[\frac{2\, y}{\left(t^2+x^2+y^2+z^2\right)^2},\, y\right]+D\left[\frac{2\, z}{\left(t^2+x^2+y^2+z^2\right)^2},\, z\right]\right)\ //\ \text{Simplify}$$

calc

0

calc

```
(D[-2 t/(t^2 + x^2 - y^2 - z^2)^2, t] + D[-2 x/(t^2 + x^2 - y^2 - z^2)^2, x] -
   D[2 y/(t^2 + x^2 - y^2 - z^2)^2, y] - D[2 z/(t^2 + x^2 - y^2 - z^2)^2, z]) // Simplify
```

calc

0

$$
\begin{aligned}
0 &= \frac{\partial \Gamma_G[\phi, \phi, t]}{\partial t} + \frac{\partial \Gamma_G[\phi, \phi, x]}{\partial x} + \frac{\partial \Gamma_G[\phi, \phi, y]}{\partial y} + \frac{\partial \Gamma_G[\phi, \phi, z]}{\partial z} \\
0 &= \frac{\partial \Gamma_G[A_x, A_x, t]}{\partial t} + \frac{\partial \Gamma_G[A_x, A_x, x]}{\partial x} - \frac{\partial \Gamma_G[A_x, A_x, y]}{\partial y} - \frac{\partial \Gamma_G[A_x, A_x, z]}{\partial z}
\end{aligned} \tag{20}
$$

Determine the Christoffel symbols for the GEM metric:

calc

```
TableForm[Partition[DeleteCases[Flatten[listaffinegem], Null], 2],
 TableSpacing -> {2, 2}]
```

calc

Γ[1, 2, 1]	$\frac{y}{(t^2-y^2-z^2)^{3/2}}$
Γ[1, 2, 2]	$-\frac{e^{\frac{2}{\sqrt{t^2-x^2-z^2}}-\frac{2}{\sqrt{t^2-y^2-z^2}}}\, x}{(t^2-x^2-z^2)^{3/2}}$
Γ[1, 3, 1]	$\frac{z}{(t^2-y^2-z^2)^{3/2}}$
Γ[1, 3, 3]	$-\frac{e^{\frac{2}{\sqrt{t^2-x^2-y^2}}-\frac{2}{\sqrt{t^2-y^2-z^2}}}\, x}{(t^2-x^2-y^2)^{3/2}}$
Γ[1, 4, 1]	$-\frac{t}{(t^2-y^2-z^2)^{3/2}}$
Γ[1, 4, 4]	$\frac{e^{-\frac{2}{\sqrt{t^2-y^2-z^2}}-\frac{2}{\sqrt{x^2+y^2+z^2}}}\, x}{(x^2+y^2+z^2)^{3/2}}$
Γ[2, 1, 1]	$-\frac{e^{-\frac{2}{\sqrt{t^2-x^2-z^2}}+\frac{2}{\sqrt{t^2-y^2-z^2}}}\, y}{(t^2-y^2-z^2)^{3/2}}$
Γ[2, 2, 1]	$\frac{x}{(t^2-x^2-z^2)^{3/2}}$
Γ[2, 3, 2]	$\frac{z}{(t^2-x^2-z^2)^{3/2}}$
Γ[2, 3, 3]	$-\frac{e^{\frac{2}{\sqrt{t^2-x^2-y^2}}-\frac{2}{\sqrt{t^2-x^2-z^2}}}\, y}{(t^2-x^2-y^2)^{3/2}}$

Γ[2, 4, 2] $-\frac{t}{\left(t^2-x^2-z^2\right)^{3/2}}$

Γ[2, 4, 4] $\frac{e^{-\frac{2}{\sqrt{t^2-x^2-z^2}}-\frac{2}{\sqrt{x^2+y^2+z^2}}}\, y}{\left(x^2+y^2+z^2\right)^{3/2}}$

Γ[3, 1, 1] $-\frac{e^{-\frac{2}{\sqrt{t^2-x^2-y^2}}+\frac{2}{\sqrt{t^2-y^2-z^2}}}\, z}{\left(t^2-y^2-z^2\right)^{3/2}}$

Γ[3, 2, 2] $-\frac{e^{-\frac{2}{\sqrt{t^2-x^2-y^2}}+\frac{2}{\sqrt{t^2-x^2-z^2}}}\, z}{\left(t^2-x^2-z^2\right)^{3/2}}$

Γ[3, 3, 1] $\frac{x}{\left(t^2-x^2-y^2\right)^{3/2}}$

Γ[3, 3, 2] $\frac{y}{\left(t^2-x^2-y^2\right)^{3/2}}$

Γ[3, 4, 3] $-\frac{t}{\left(t^2-x^2-y^2\right)^{3/2}}$

Γ[3, 4, 4] $\frac{e^{-\frac{2}{\sqrt{t^2-x^2-y^2}}-\frac{2}{\sqrt{x^2+y^2+z^2}}}\, z}{\left(x^2+y^2+z^2\right)^{3/2}}$

Γ[4, 1, 1] $-\frac{e^{\frac{2}{\sqrt{t^2-y^2-z^2}}+\frac{2}{\sqrt{x^2+y^2+z^2}}}\, t}{\left(t^2-y^2-z^2\right)^{3/2}}$

Γ[4, 2, 2] $-\frac{e^{\frac{2}{\sqrt{t^2-x^2-z^2}}+\frac{2}{\sqrt{x^2+y^2+z^2}}}\, t}{\left(t^2-x^2-z^2\right)^{3/2}}$

Γ[4, 3, 3] $-\frac{e^{\frac{2}{\sqrt{t^2-x^2-y^2}}+\frac{2}{\sqrt{x^2+y^2+z^2}}}\, t}{\left(t^2-x^2-y^2\right)^{3/2}}$

Γ[4, 4, 1] $\frac{x}{\left(x^2+y^2+z^2\right)^{3/2}}$

Γ[4, 4, 2] $\frac{y}{\left(x^2+y^2+z^2\right)^{3/2}}$

Γ[4, 4, 3] $\frac{z}{\left(x^2+y^2+z^2\right)^{3/2}}$

"Γ[1, 2, 1]" $\frac{y}{\left(t^2-y^2-z^2\right)^{3/2}}$

"Γ[1, 2, 2]" $-\frac{e^{\frac{2}{\sqrt{t^2-x^2-z^2}}-\frac{2}{\sqrt{t^2-y^2-z^2}}}\, x}{\left(t^2-x^2-z^2\right)^{3/2}}$

"Γ[1, 3, 1]" $\frac{z}{\left(t^2-y^2-z^2\right)^{3/2}}$

"Γ[1, 3, 3]"	$-\frac{e^{\frac{2}{\sqrt{t^2-x^2-y^2}}-\frac{2}{\sqrt{t^2-y^2-z^2}}}\,x}{\left(t^2-x^2-y^2\right)^{3/2}}$
"Γ[1, 4, 1]"	$-\frac{t}{\left(t^2-y^2-z^2\right)^{3/2}}$
"Γ[1, 4, 4]"	$\frac{e^{-\frac{2}{\sqrt{t^2-y^2-z^2}}-\frac{2}{\sqrt{x^2+y^2+z^2}}}\,x}{\left(x^2+y^2+z^2\right)^{3/2}}$
"Γ[2, 1, 1]"	$-\frac{e^{-\frac{2}{\sqrt{t^2-x^2-z^2}}+\frac{2}{\sqrt{t^2-y^2-z^2}}}\,y}{\left(t^2-y^2-z^2\right)^{3/2}}$
"Γ[2, 2, 1]"	$\frac{x}{\left(t^2-x^2-z^2\right)^{3/2}}$
"Γ[2, 3, 2]"	$\frac{z}{\left(t^2-x^2-z^2\right)^{3/2}}$
"Γ[2, 3, 3]"	$-\frac{e^{\frac{2}{\sqrt{t^2-x^2-y^2}}-\frac{2}{\sqrt{t^2-x^2-z^2}}}\,y}{\left(t^2-x^2-y^2\right)^{3/2}}$
"Γ[2, 4, 2]"	$-\frac{t}{\left(t^2-x^2-z^2\right)^{3/2}}$
"Γ[2, 4, 4]"	$\frac{e^{-\frac{2}{\sqrt{t^2-x^2-z^2}}-\frac{2}{\sqrt{x^2+y^2+z^2}}}\,y}{\left(x^2+y^2+z^2\right)^{3/2}}$
"Γ[3, 1, 1]"	$-\frac{e^{-\frac{2}{\sqrt{t^2-x^2-y^2}}+\frac{2}{\sqrt{t^2-y^2-z^2}}}\,z}{\left(t^2-y^2-z^2\right)^{3/2}}$
"Γ[3, 2, 2]"	$-\frac{e^{-\frac{2}{\sqrt{t^2-x^2-y^2}}+\frac{2}{\sqrt{t^2-x^2-z^2}}}\,z}{\left(t^2-x^2-z^2\right)^{3/2}}$
"Γ[3, 3, 1]"	$\frac{x}{\left(t^2-x^2-y^2\right)^{3/2}}$
"Γ[3, 3, 2]"	$\frac{y}{\left(t^2-x^2-y^2\right)^{3/2}}$
"Γ[3, 4, 3]"	$-\frac{t}{\left(t^2-x^2-y^2\right)^{3/2}}$
"Γ[3, 4, 4]"	$\frac{e^{-\frac{2}{\sqrt{t^2-x^2-y^2}}-\frac{2}{\sqrt{x^2+y^2+z^2}}}\,z}{\left(x^2+y^2+z^2\right)^{3/2}}$
"Γ[4, 1, 1]"	$-\frac{e^{\frac{2}{\sqrt{t^2-y^2-z^2}}+\frac{2}{\sqrt{x^2+y^2+z^2}}}\,t}{\left(t^2-y^2-z^2\right)^{3/2}}$

"Γ[4, 2, 2]"	$-\frac{e^{\frac{2}{\sqrt{t^2-x^2-z^2}}+\frac{2}{\sqrt{x^2+y^2+z^2}}}\,t}{\left(t^2-x^2-z^2\right)^{3/2}}$
"Γ[4, 3, 3]"	$-\frac{e^{\frac{2}{\sqrt{t^2-x^2-y^2}}+\frac{2}{\sqrt{x^2+y^2+z^2}}}\,t}{\left(t^2-x^2-y^2\right)^{3/2}}$
"Γ[4, 4, 1]"	$\frac{x}{\left(x^2+y^2+z^2\right)^{3/2}}$
"Γ[4, 4, 2]"	$\frac{y}{\left(x^2+y^2+z^2\right)^{3/2}}$
"Γ[4, 4, 3]"	$\frac{z}{\left(x^2+y^2+z^2\right)^{3/2}}$

Again there are many exponential terms that will make life a challenge. Pick out the double-potentials that are simpler:

calc

$$\left(D\left[\frac{2x}{\left(x^2+y^2+z^2\right)^{3/2}}, x\right] + D\left[\frac{2y}{\left(x^2+y^2+z^2\right)^{3/2}}, y\right] + D\left[\frac{2z}{\left(x^2+y^2+z^2\right)^{3/2}}, z\right]\right) \text{ // Simplify}$$

calc

0

calc

$$\left(D\left[-\frac{t}{\left(t^2-y^2-z^2\right)^{3/2}}, t\right] - D\left[\frac{y}{\left(t^2-y^2-z^2\right)^{3/2}}, y\right] - D\left[\frac{z}{\left(t^2-y^2-z^2\right)^{3/2}}, z\right]\right) \text{ // Simplify}$$

calc

0

$$\begin{aligned} 0 &= \frac{\partial\Gamma_G[\phi, \phi, x]}{\partial x} + \frac{\partial\Gamma_G[\phi, \phi, y]}{\partial y} + \frac{\partial\Gamma_G[\phi, \phi, z]}{\partial z} \\ 0 &= \frac{\partial\Gamma_G[A_x, A_x, t]}{\partial t} - \frac{\partial\Gamma_G[A_x, A_x, y]}{\partial y} - \frac{\partial\Gamma_G[A_x, A_x, z]}{\partial z} \end{aligned} \tag{22}$$

This strikes me as a non-trivial link between potentials and metrics.

Speculations

The metrics written here just don't work to solve the 4D field equations for EM, hypercomplex gravity, or the GEM field equations. There is no honest way to defend wrong.

Riemann geometry can work in an arbitrary number of dimensions, it is a power tool. That may be the source of the problem. Riemann geometry specifically does not come equipped with the ability to multiply and divide, the real power tools of math. If one stays confined to 3 vector dimensions of space and 1 scalar dimension of time like the three field equations in question, there may be a different way to account for changes in spacetime basis vectors. The [possibly] interesting failure of this notebook will motivate a different approach to describing curvature.

BIG WARNING: a similar section in my big paper (QMN:1009.9466) might be wrong. I may well have ignored terms I didn't want. Either that or I am not handling Christoffel symbols correctly. Tough stuff, Christoffel symbols.

▪ Tools

$$\text{Units} = \left\{G \to \frac{L^3}{M\,T^2},\ C \to \frac{L}{T},\ \hbar \to \frac{M\,L^2}{T},\ m \to M,\ q \to \frac{\sqrt{M\,L^3}}{T}\right\};$$